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Leptonic CP violation and neutrino mass in a three generation $SU(2)_H$ gauge model

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Abstract : We have shown that with suitable Higgs field the lepton mass matrix in $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$ gauge model is identical with the mass matrix *ansatz* based on the permutation symmetry. The possibility of leptonic CP violation is discussed in terms of CP-violating rephasing-invariant quantity J^l_{CP}. The neutrino mass bounds are also mentioned on the basis of the mass matrix obtained.

Keywords: CP Violation, neutrino mass, horizontal symmetry.

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1. Introduction

Although the Standard Electro-weak model [1] correctly describes weak interaction phenomenology at presently available energies, but the quark and lepton masses and flavour mixing angles are not predictable. Within the standard model, all masses and flavour mixing angles are free parameters and no relations among them are provided. Perhaps, a new theory could predict all masses and flavour mixing parameters in terms of some new, few fundamental parameters. Recent neutrino experimental results and cosmological observations [2] show evidence for non-zero neutrino mass and provide possibility of lepton flavor mixing. The standard model then has to be enlarged and we have more free parameters to describe the fermion masses and mixing angles. Although non-zero neutrino masses and mixing can be interpreted as the solution to the solar [3] and atmospheric [4] neutrino anomalies, the present experiments do not provide the values of neutrino masses and mixing angles in three neutrino oscillation scheme. The vacuum oscillation and small angle MSW solutions of solar neutrino data are also disfavoured by the current data. The solution for solar neutrino deficit may be either small or large mixing with different mass squared differences depending on MSW effect [5] and hence, we can atmost estimate the neutrino mass hierarchy and their mixing in extended gauge model. In this context, horizontal interactions among the fermion families have been introduced to restrict the independent parameters of fermion mass matrix and to obtain some insight on the problems of fermion family repetition. Several horizontal symmetries have been introduced by different authors [6] to understand the generation problem of fermions including their masses, mixing angles and CP violation.

In the present work, we focus our attention on neutrino mass, lepton mixing as well as leptonic CP violation in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$ horizontal gauge model. Our analysis shows that in the three generation left-right symmetric $SU(2)_H$ model, the neutrino mass matrix is identical with the mass matrix *ansatz* based on the permutation symmetry with the suitable choice of Higgs

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field. The CP violating phase in three generation $SU(2)_H$ model lies in the range $0 < \delta < \pi/2$ where $\delta_{13} = \delta$ is considered.The value of J^l_{cp} is approximately equal to 1.3×10^{-3} [2] and it is small compared to large mixing angle (LMA) MSW solution or vacuum oscillation (VO) solution. The bounds on neutrino masses are also discussed and the results are consistent with the recent experiments [7].

The plan of the paper is as follows. In Section 2, we present our model and the parameters for lepton mass matrices entering in the model and leptonic CP violation also. In Section 3, we summarize our analysis. Section 4 contains our conclusions.

2. Lepton mass matrices and leptonic CP-violation

We consider a three generation left-right symmetric model with $SU(2)_H$ as the horizontal symmetry [6] based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$. Besides their usual representations under $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the left-and right-handed fermions under $SU(2)_H$ transform as triplets for the three generation model. The representation contents of the fermions as well as Higgs field with their VEV's under $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$ are :

$$\psi_L(2,1,-1,3) = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e^- & \mu^- & \tau^- \end{pmatrix}, \tag{1}$$

$$\psi_R(1,2,-1,3) = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e^- & \mu^- & \tau^- \end{pmatrix}. \tag{2}$$

VEV's of the Higgs fields are :

$$< \phi_{ij} > (2,2,0,5) = \begin{pmatrix} k_{ij}\exp(i\theta_{ij}) & 0 \\ 0 & k'_{ij}\exp(i\theta'_{ij}) \end{pmatrix},$$

$$i, j = 1, 2,3 \tag{3}$$

$$< \chi_{ij} > (2,2,0,3) = \begin{pmatrix} p_{ij}\exp(i\alpha_{ij}) & 0 \\ 0 & p'_{ij}\exp(i\alpha'_{ij}) \end{pmatrix}, \tag{4}$$

$$< \eta > (1,1,0,3) = h_k \exp(i\beta_k), \tag{5}$$

$$< \Delta_L > (3,1,2,1) = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \tag{6}$$

$$< \Delta_R > (1,3,2,1) = \begin{pmatrix} 0 & 0 \\ v_k & 0 \end{pmatrix}, \tag{7}$$

We have chosen the minimal set of Higgs fields where ϕ and χ generate masses of the basic fermions and the left-and right-handed and horizontal gauge bosons, η gives masses to the horizontal gauge bosons and Δ_L, Δ_R are introduced to break the left-right symmetry.

The most general renormalizable Higgs potential of the model is

$$V = V_\phi + V_\chi + V_\eta + V_\Delta + V_{\phi\Delta} + V_{\chi\Delta} + V_{\eta\Delta} + V_{\chi\eta} + V_{\phi\chi} \tag{8}$$

The minimization of the Higgs potential leads to

$$k_{11} = 0,\ k_{12} \neq 0,\ k_{13} = 0,\ k_{22} = 0,\ k_{23} \neq 0,\ k_{33} = 0,$$
$$k'_{11} = 0,\ k'_{12} \neq 0,\ k'_{13} = 0,\ k'_{22} = 0,\ k'_{23} \neq 0,\ k'_{33} = 0, \tag{9}$$
$$p_{11} = 0,\ p_{22} \neq 0,\ p_{33} \neq 0,\ p'_{11} = 0,\ p'_{22} \neq 0,\ p'_{33} \neq 0,$$
$$\theta_{ij} = \theta'_{ij},$$

and $\alpha_{ij} = \alpha'_{ij}$.

The most general and basic lepton-Higgs interaction is

$$L = f_1\ \bar{\psi}^i\alpha_L\psi^j\beta_R\phi^{ij}\alpha\beta + f_2\bar{\psi}^i\alpha_L\psi^j\beta_R\ \chi^{ij}\alpha\beta + \text{h.c.} \tag{10}$$

$$ij = 1, 2, 3 \text{ and } \alpha, \beta = 1, 2$$

The couplings f_1 and f_2 are independent of $SU(2)_L$, $SU(2)_R$, $SU(2)_H$ indices and hence, they are real numbers. In this model, we obtain the mass matrices for both neutrinos and charged leptons, which have the following form

$$M^\nu = \begin{pmatrix} 0 & a_1 & 0 \\ a_1 & b_1 & c_1 \\ 0 & c_1 & d_1 \end{pmatrix} \text{ and } M^l = \begin{pmatrix} 0 & a_2 & 0 \\ a_2 & b_2 & c_2 \\ 0 & c_2 & d_2 \end{pmatrix}. \tag{11}$$

where $a_1 = k_{12}\exp(i\theta_{12})$, $b_1 = p_{22}\exp(i\alpha_{22})$,

$c_1 = k_{23}\exp(i\theta_{23})$,

$d_1 = p_{33}\exp(i\alpha_{33})$, $a_2 = k'_{12}\exp(i\theta_{12})$,

$b_2 = p'_{22}\exp(i\alpha_{22})$,

$c_2 = k'_{23}\exp(i\theta_{23})$, $d_2 = p'_{33}\exp(i\alpha_{33})$,

The parameters a_1, b_1, c_1, d_1, and a_2, b_2, c_2, d_2 can be expressed in terms of fermions mass eigen values and one free parameter ε^1. It is seen that even if the phases α_{22} and α_{33} are removed by redefining the fermion fields, no change in the results will occur. The parameters can be expressed as follows :

$$a_1 = \sqrt{2m_1m_2},\ b_1 = m_3/2\left[1 + 2.\frac{m_2 - m_1}{m_3}\right.$$

$$c_1 = m_3/2\left|1-\frac{m_2-m_1}{m_3}\right|,\ d_1 = m_3/2$$

and

$$a_2 = \sqrt{m_e m_\tau},\ b_2 = m_\mu,\ c_2 = \sqrt{\varepsilon^1 m_\tau},\ d_2 = m_\tau .$$

The expressions for a_2, b_2, c_2, d_2 are written considering the fact that the charged lepton family has the mass hierarchy $m_e << m_\mu << m_\tau$. We can take the mass eigen values m_1, m_2, and m_3 to get the fermion mass matrix. The mass matrices for charged leptons and neutrinos have the same form and the free parameter ε^1 is taken to be identical in both the charged lepton and neutrino mass matrices and can be determined from the neutrino experimental results.

The mass matrices are complex symmetric in both the sectors of leptons. The lepton flavour-mixing matrix in the standard parameterization can be given as

$$V^1_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\exp(-i\delta_{13}) \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\exp(i\delta_{13}) & c_{12}c_{23} - s_{12}s_{23}s_{13}\exp(i\delta_{13}) & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\exp(i\delta_{13}) & -c_{12}s_{23} - s_{12}c_{23}s_{13}\exp(i\delta_{13}) & c_{23}c_{13} \end{pmatrix},$$

where $s_{ij} = \sin\theta_{ij}$ and $c_{ij} = \cos\theta_{ij}$ $(i, j = 1, 2, 3)$.

The mixing angles θ_{ij} can be expressed in terms of masses of the leptons and after constraining [2] the lepton mixing matrix will take the approximate form as

$$V^1_{\rm CKM} = \begin{pmatrix} c_{12} & s_{12} & s_{13}\exp(-i\delta_{13}) \\ -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\ s_{12}s_{23} & -c_{12}s_{23} & c_{23} \end{pmatrix}.$$

It is to be noted that the mixing matrix contains non-zero CP-violating phase. The value of CP-violating rephasing-invariant quantity $J_{CP} \approx 1.3 \times 10^{-3}$ for $0 \le \delta_{13} \le \pi/2$. For $\pi/2 < \delta_{13} < \pi$, the value of J^l_{CP} is much more suppressed. However, compared to quark sector result J^l_{CP} is large and hence, lepton number violating CP violation may be indeed large.

The form of the mass matrix provides [2] numerical values of the neutrino mass eigen values, although they are not exact. But some possible ranges can be estimated. If we assume the mass hierarchy of m_1, $m_2 << m_3$, the value of $m_3 \approx 4.7 \times 10^{-2}$ eV in case of atmospheric neutrino. In large mixing angle (LMA) solution, $m_1 \approx m_2$ and the allowed neutrino mass bounds are : $3.0 \times 10^{-4} \le m_1 \le 2.0 \times 10^{-3}$ eV and $2.7 \times 10^{-3} \le m_2 \le 1.5 \times 10^{-2}$ eV for $5 \times 10^{-6} \le \Delta m^2_{\rm solar} \le 4 \times 10^{-5}$ eV^2 with $\theta_{\rm solar} \approx$ arc tan (m_1/m_2). With the mass hierarchy m_1, $m_2 << m_3$, $1.3 \times 10^{-2} \le \varepsilon^1 \le 2.4 \times 10^{-2}$ eV. For large mixing between ν_μ and ν_τ, $\varepsilon^1 \approx m_3/2$ and m_1, $m_2 << m_3$ [2] In case of vacuum oscillation (VO) solution, the lower bounds of neutrino masses are $m_1 \ge 0.24 \times 10^{-5}$ eV, $m_2 \ge 0.93 \times 10^{-5}$ eV. The mixing angle θ_{13} can also be determined using m_1 and m_2 [2]. In case of small mixing angle (SMA) solution, m_1 may have small mixing angle value without a lower bound, whereas the upper bound depends upon CP violating phase δ_{13} and it ranges from, 3.0×10^{-7} eV to 4.2×10^{-5} eV. The lower mass bound of m_2 is $\approx 1.9 \times 10^{-3}$ eV [2].

3. Summary

We have studied neutrino masses and mixing in the three generation left-right symmetric horizontal gauge model with $SU(2)_H$ as horizontal symmetry. The mass matrix we obtained after minimizing the potential containing minimal Higgs field, is similar to the mass matrix *ansatz* as given in Ref. [2]. The model suggests the mass hierarchy $m_1 \approx m_2 << m_3$ to support the LMA and VO solution and also the hierarchy $m_1 << m_2 << m_3$ for SMA solution. Although the vacuum oscillation and small mixing MSW solutions of solar neutrino data are disfavoured by current data but our motivation to show leptonic CP violation is still favoured because of non-violating rephrasing invariant quantity J^l_{CP}. The magnitude of CP-violating rephrasing-invariant quantity J^l_{CP} depends on the CP-violating phase angle and the value may be upto 1.3×10^{-3}.

4. Conclusion

We thus conclude that extended gauge model with $SU(2)_H$ horizontal gauge group for three generations of leptons may give rise to leptonic CP-violation which may be large compared to that in the quark sector. Furthermore, the model gives rise to the mass matrix *ansatz*, which supports the possible bounds on neutrino masses also. Thus, $SU(2)_H$ gauge symmetry contributes to non-zero neutrino mass unlike the standard model and also excludes the heavy right-handed neutrino. It is therefore, interesting to study the other horizontal gauge group also [8].

References

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